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RESEARCH ON DYNAMICS OF COMPOSITE AND SANDWICH PLATES, 1979-81.(U)
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RESEARCH ON DYNAMICS OF COMPOSITE AND
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by

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RESEARCH ON DYNAMICS OF COMPOSITE
AND SANDWICH PLATES, 1979-81

C.W. Bert*

*Abstract - This paper presents a survey of the literature concerning dynamics of plate-type structural elements of either composite material or sandwich construction. Papers from mid-1979** through early 1982 are reviewed. Particular attention is given to experimental research and to linear and nonlinear analysis. Configurations include rectangularly orthotropic, cylindrically orthotropic, and anisotropic plates; laminated plates; and thick and sandwich plates. Free and forced vibration, flutter, and impact are considered.*

1. INTRODUCTION

The fundamentals of the mechanics of composite and laminated plates have been discussed in a previous two-part survey [1], which was updated in 1979 [2]. Other recent surveys that are closely related to the present work include the present author's survey of vibration of composite structures [3], one by Leissa on complicating effects in free vibration of plates [4], another by the same author on vibration, buckling, and postbuckling of composite plates [5], and one by Reddy on finite-element analyses of composite plates and shells [6].

Information sources referenced in this survey are primarily papers in

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**A few 1978 and early 1979 references unavoidably omitted in [2] are also included.



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the open literature and a few additional reports. The following topics are not included: anisotropic-crystal plates, magnetoelastic effects, and plates with cracks. Since it is not possible to include all of the work published in the time frame covered, the author apologizes to persons whose work may have been inadvertently omitted.

2. EXPERIMENTAL RESEARCH

Here, we discuss research which is primarily experimental, rather than analytical, in nature. It is very encouraging to note the increased activity in this category of research since the 1970 survey [2].

2.1 Sinusoidal and Random Loadings

Crawley [7] investigated the resonant frequencies and mode shapes of a series of clamped rectangular plates and cylindrically curved panels. The plates were of graphite/epoxy and hybrid graphite/epoxy/aluminum alloy, laminated in various symmetric lamination schemes. The experimental results were compared with those of a mixed finite element developed by Lee and Pian [8]. Agreement for mode shape was excellent while that for frequencies was reasonable, with discrepancies attributed to differences between dynamic flexural and static in-plane moduli.

Rasskasov and Sokolovskaya [9] investigated the static and frequency response of a series of hinged and combined-support rectangular plates of single-core and double-core sandwich construction. They included four lamination schemes symmetric with respect to the midplane and four unsymmetric. The stiff layers were of sheet steel and of glass/polymer, while the cores were of foam plastic. The results were compared with analytical results for hinged supports.

Teh and White [10] conducted experiments on eight-layer graphite/epoxy

(and aluminum-alloy) panels with clamped edges. To simulate conditions representative of an aircraft flight environment, the panels were subjected to a combination of uniaxial compression and random acoustic pressure loading. Experimentally determined frequencies were generally lower than those obtained by Rayleigh-Ritz theoretical analysis.

Chamis and his associates [11] reported on a very innovative series of experiments to determine the resonant frequencies and damping factors of laminated panels previously subjected to damage induced by residual stresses and monotonic or cyclic applied loads. The materials were glass/epoxy and high-modulus-graphite/epoxy in various lamination schemes. It was concluded that the dynamic response of the glass/epoxy panels was susceptible to low-level damage, while that of the graphite/epoxy ones was not.

2.2 Impact and Blast Loading

There has been considerable recent experimental research on impact loading of composite plates, as surveyed recently by Takeda and Sierakowski [12]. First should be mentioned the extensive series of experiments reported by Takeda, Sierakowski, and their associates [13-16]. These were all concerned with glass/epoxy laminated panels subjected to local ballistic impact, with emphasis on delamination failure mechanisms.

Rhodes and his associates [17] investigated low-velocity impact damage in graphite/epoxy panels, while Hayes and Rybicki [18] reported on experiments on panels of graphite/epoxy, aramid/epoxy, and their hybrids. They also concentrated on delamination failure. Knauss [19] reported on the use of the moire fringe technique to determine the phenomenological aspects of damage due to low-velocity impact in graphite/epoxy laminates.

In a series of experiments, C.T. Sun of Purdue University and his

associates [20-22] focussed attention on the contact law and its role in the dynamic response of locally impacted plates.

Rajamani and Prabhakaran [23] investigated the response of unidirectional glass-epoxy plates subjected to blast loading produced by a shock tube. Both solid plates and plates with a central circular hole were studied. It is noted that the measured dynamic amplification factors for glass/epoxy averaged about 35% lower than those for homogeneous aluminum-alloy plates.

3. LINEAR ANALYSES OF THIN PLATES

These analyses assume linear stress-strain behavior of the material and small deflections so that the strain-displacement relations are linear. Furthermore, in many cases (Sections 3.1-3.3), the material is assumed to be macroscopically homogeneous through the thickness. Thus, the material may consist of either a single layer of composite material, or multiple layers provided that they all have the same orientation. Three categories of reinforcement geometry are considered: specially orthotropic (Section 3.1) and generally orthotropic (equivalent to anisotropic) with respect to rectilinear coordinates (Section 3.2), and cylindrically orthotropic (such as approximated by manufacture using the filament-winding process) in Section 3.3. Thin laminates (nonhomogeneous through the thickness) are discussed in Section 3.4.

3.1 Specially Orthotropic Thin Plates

Such plates have the principal-material-symmetry axis oriented parallel to a geometric axis of the plate (such as a center line or axis of symmetry). Sakata [24,25] reviewed the use of reduction methods to convert numerical results for isotropic plates to those for specially orthotropic plates (see Refs. 1-5 in [2]).

In a series of papers Laura and his associates [26-31] used polynomial approximating functions in conjunction with either the Rayleigh-Ritz or the Galerkin method. References [26-28] were concerned with rectangular plates having elastically restrained edges: [26] analyzed various combinations of free and elastic edges; [27] had attached concentrated mass; and [28] treated plates tapered in two directions. Reference [29] treated the solid circular planform; [30] considered polygonal planform; and [31] presented a methodology for analysis of either clamped or simply supported planform (with numerical results only for regular polygons).

A number of other analyses were concerned with rectangular-planform plates. For example, Wilson [32] considered an infinite plate strip on an elastic foundation and subjected to line loads and moments traveling at constant speed. It is cautioned that in this analysis, an incorrect equivalent isotropic plate approach is used. For example, the Poisson-bending and twisting term

$$2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2}$$

is replaced by the following much more restrictive term:

$$2(D_{11}D_{22})^{1/2} \frac{\partial^4 w}{\partial x^2 \partial y^2}$$

where D_{11} and D_{22} are the flexural rigidities in the x and y directions, D_{12} is the Poisson-bending rigidity, D_{66} is the twisting rigidity, w is the plate deflection, and x and y are the longitudinal and transverse directions in the plane of the plate.

Sakata [33] treated simply supported rectangular plates with stepped thicknesses, using the reduction method (see [24,25]). Ganesan and Dhotarad

[34] analyzed plates tapered in one direction, with temperature-dependent elastic properties, and subjected to a temperature gradient. Using the finite-difference method, they considered all edges clamped, all edges simply supported, and opposite edges clamped and simply supported. Sobotka [35] analyzed viscoelastic plates with the latter two combinations of boundary conditions. However, this analysis has been severely criticized in [36].

Kuttler and Siglillito [37] applied to clamped rectangular plates their method [38] for obtaining upper and lower bounds on the natural frequencies. Bucco et al. [39] applied a combination of the finite-strip method [40] and the deflection-contour method [41] to various shapes of isotropic plates and to clamped, square, orthotropic plates. Narita [42] used a series method to attack the problem of free vibration of a plate that is partially restrained along portions of its edges and simply supported on the remainder.

Simply supported plates of parallelogram planform were considered by Sakata [43] using the reduction method (see [24,25]). Forced vibration of polygonal plates with linear damping was analyzed by Katsaitis [44]. Plates of infinite planform extent were considered by Busch-Vishniac [45], who derived the driving-point impedance in the presence of initial tension, and by Das and Roy [46], who considered arbitrary forcing functions for plates on elastic foundations.

3.2 Anisotropic (Generally Orthotropic) Thin Plates

In a series of papers [47-49], Laura and his associates considered plates of rectangular planform, using polynomial approximating functions and either the Rayleigh-Ritz or the Galerkin method. In [47], elastically restrained edges were treated, and in [48], clamped edges and in-plane initial loads were included. In [49], the effect of a small, free-edge diagonal cutout at a corner was investigated.

Sakata and Hayashi [50] conducted both analytical (using the reduction method) and experimental investigations of parallelogram plates.

Irie and Yamada [51] used the Rayleigh-Ritz method, in conjunction with spline functions, to analyze elliptic plates with confocal elliptic holes. Although the equations were developed for general orthotropy, they were implemented numerically only for special orthotropy.

3.3 Polar (Cylindrically) Orthotropic Thin Plates

Cylindrically orthotropic (or polar orthotropic) plates are those which have directions of material symmetry that coincide with a circular cylindrical coordinate system. In practice, they are made by winding filaments circumferentially on a circular mandrel. In a plate (or disk) configuration, they are most widely used as reinforcements at circular cutouts or as rotating disks (energy-storage flywheels, and turbine or compressor disks). In both of these categories of application, biaxial loadings are present and the most efficient designs involve varying thickness (radial taper). Surprisingly, during the time interval covered in this survey, apparently only one paper on vibration of a varying thickness plate subjected to in-plane preload appeared. This was the work of Dyka and Carney [52], in which they considered a circular annular plate with parabolic thickness variation and ring-type stiffeners at both edges, and subjected to uniform radial compressive load at the outer edge.

Circular plates subjected to centrifugal preload were considered by several investigators. The in-plane torsional vibrations of both radially tapered and uniform-thickness disks were analyzed by Ochan [53], who developed expressions for the critical speed of rotation for dynamic instability. Flexural vibration of a uniform-thickness plate with ring-type stiffeners at both edges was treated by Dyka and Carney [54]. Laura et al.

[55] made Rayleigh-Ritz and FEM (finite-element method) analyses of uniform plates subjected to uniform in-plane prestress.

Flexural vibration of varying thickness plates without preload were studied by several investigators. Lenox and Conway [56] presented a closed-form solution for vibration of a plate with a parabolic thickness variation vibrating in any arbitrary combination of radial and circumferential nodes. This solution should be extremely valuable to assess the accuracy of various approximate numerical techniques. Bell and Kirkhope [57] considered plates with piecewise stepped radial thickness variation by extending the isotropic transfer-matrix analysis due to Ehrich [58].

Both FEM analysis and experiments were used by Ginesu et al. [59] to study flexural vibrations of uniform-thickness plates with various boundary conditions. Gorman [60] used the FEM analysis of [59] to develop extensive tabular results for uniform-thickness annular plates with various combinations of clamped, free, and simply supported edge conditions. Avalos and Laura [61] used a Galerkin polynomial approach to treat the axisymmetric flexural modes of annular plates with elastic rotational restraints. Tani [62] investigated the dynamic instability of annular plates subjected to pulsating torsion in their plane.

Sector plates have a quadrilateral planform in the shape of a sector of an annular circular plate. The Rayleigh-Ritz method was used to analyze free vibrations of such plates. Irie et al. [63] used spline functions as the admissible function, while Ramaiah [64] used simple polynomials.

3.4 Laminated Thin Plates

In the time frame covered in this survey, there have been relatively few linear analyses of thin laminated plates. C.T. Sun of the University

of Florida presented an excellent tutorial exposition [65]. Crawley and Dujundji [66] made a Kantorovich variational analysis of symmetrically laminated cantilever plates. Lin [67] considered clamped, free, and simply supported plates resting on a viscoelastic foundation and subjected to a constant force moving along the plate at constant speed.

Stavsky and his associates [68,69] considered the arbitrary-mode vibration of circular plates consisting of concentric cylindrically orthotropic layers. Two different eigenvalue solution schemes were used: in [68] a classical approach was used, while in [69] a finite-difference scheme was applied. In these works, it was shown that certain lamination schemes are capable of producing higher fundamental frequencies that can be attained by homogeneous plates of either constituent material. This is not surprising, as it is the basis for the use of sandwich construction (see Section 4.3).

Dynamic fracture mechanics of a laminated plate was considered in [70]. Laminated composite plates (cross-ply laminates and isotropic-material laminates) with discrete stiffeners were analyzed by Chao and Lee [71] using a classical approach.

Although the potential for tailoring of laminates is usually mentioned as one of their advantages, in practice to date, most optimization has been on an ad hoc basis and has not considered dynamic criteria (objective functions or constraints). A paper which attempts to remedy this situation is the recent formal optimization, using nonlinear mathematical programming, by Rao and Singh [72]. They considered minimum-weight design with constraints on minimum fundamental natural frequency, minimum buckling load, and maximum static deflection.

4. LINEAR ANALYSES OF MODERATELY THICK PLATES

Due to the low thickness shear moduli of fiber-reinforced composite materials relative to their flexural elastic moduli, it is advisable to include thickness shear deformations (sometimes called transverse shear deformations) in dynamic analyses of plates made of such materials. This is often necessary even in the case of plates having geometrical parameters such that they would be considered thin if they were constructed of homogeneous isotropic material. This motivation, which was pointed out in [1,2], apparently has resulted in the generation of a number of recent analyses, discussed in Section 4.1 and especially in Section 4.2. Since there is considerable analogy between so-called sandwich plates and homogeneous or laminated plates with thickness shear deformation included, sandwich plates are discussed in Section 4.3.

4.1 Specially Orthotropic, Generally Orthotropic, and Cylindrically Orthotropic Plates with Thickness Shear Flexibility

Kuznetsov et al. [73] presented a new, improved theory which includes thickness stretching as well as thickness shearing deformation. They applied their new theory in a Ritz-Galerkin analysis of a cantilever, rectangular plate of CFRP (carbon-fiber reinforced plastic) and hybrid glass FRP and CFRP plates.

Dobyns [74] presented a simplified analysis of simply supported plates subjected to blast loading. Actually he called his materials laminates, but neglected bending-stretching coupling (B_{ij}) and bending-twisting coupling (D_{16} and D_{26}). The symbols are classical laminated plate theory notation (cf. [75] or [76]). Thangam Babu et al. [77] extended the Ventakeswara Rao et al. orthotropic high-precision finite element [78] by adding the effects of an elastic foundation.

Patra and Iyengar [79] made a displacement-function FEM analysis of

a generally orthotropic rectangular plate clamped at its outer edges and containing a free-edge circular or rectangular cutout.

Cheung and Chan [80] applied the finite-strip method (see [40]) to cylindrically orthotropic sectorial plates (see also [63] and [64]).

4.2 Laminated Plates with Thickness Shear Flexibility

In a recent monograph, Bolotin and Novichkov [81] covered many different aspects of the mechanics of laminates. Chapter 7 is concerned with vibration and wave propagation of laminated plates including thickness shear deformation.

Green and Naghdi [82] developed a new dynamic thermoelastic theory of plates laminated of orthotropic materials and applied it to analysis of propagation of harmonic waves in three-layer plates. Khoroshun [83] also developed a new theory of laminated plates and shells which differs from the classical Reissner-Mindlin type theories (and Timoshenko beam theory) in that it does not require ad hoc determination of the shear correction factors. In this respect it is analogous to the new theory of homogeneous plates introduced by Levinson [84]. Khoroshun and Ivanov [85] applied Khoroshun's new theory to wave propagation in two-layer laminated plates.

Chatterjee and Kulkarni [86] developed a dynamic approach to determination of the shear correction factors for the Whitney-Pagano shear-deformable laminate theory [87]. These were obtained by matching cutoff frequencies for propagation of thickness shear waves predicted by plate and elasticity theories. In general they result in slightly lower values than those predicted by Whitney [88] using a static approach. Chatterjee and Kulkarni [89] made an extensive investigation of the effects of material damping, temperature, and moisture on the panel flutter of graphite/epoxy laminates.

The moving-load response of a two-layer plate on a compressible fluid

half-space was investigated by Chonan [90]. A similar configuration was studied by Crighton [91].

Reddy [92] used his isoparametric finite element to analyze free vibration of simply supported rectangular plates of angle-ply lamination scheme. He gave numerical results showing the effects of plate aspect ratio, relative thickness, and lamination angle on plates of two different materials typical of high-modulus graphite/epoxy and high-strength graphite/epoxy. Reddy and Chao [93] studied the effect of reduced integration, mesh size, and element type (linear or quadratic) on predicted natural frequencies of cross-ply and angle-ply plates.

Witt and Sobczyk [94] analyzed the cylindrical-bending response of simply supported laminated plates to random-pressure loading. Sih and Chen [95] performed a dynamic fracture mechanics analysis of a four-layer plate containing a crack and subjected to sudden stretching. In a series of three papers, Guyader and Lesueur [96-98] considered the vibration modes, transmission under oblique plane-wave excitation, and transmission under reverberant sound excitation.

Wave propagation in laminated plates was recently considered by two different sets of investigators. Kim and Moon [99] used a Laplace transform in time and a Fourier transform in space and reported information on longitudinal waves, thickness waves, and wavefront surfaces. A similar analysis was made by Sun and Tan [100], who then tied it in with Sun's previous impact-law work [20-22] to predict plate response to localized impact loading.

Certain materials, including fiber-reinforced composites with soft matrices, biological tissues, and brittle materials such as concrete, have quite different stress-strain curves when loaded in compression rather than

tension. As a first approximation to the stress-strain behavior of such materials, Timoshenko [101] (one dimensional case) and later Ambartsumyan [102] (general case) suggested the use of a bilinear approximation, with different moduli in tension and compression (thus called bimodular). This approach was extended to fiber-governed soft-matrix composites in [103]. Recently Bert, Reddy, et al. [104] presented the results of closed-form and FEM analyses for free vibration of rectangular plates cross-plyed of such bimodular composite materials. More recently Reddy [105] reported on FEM analyses of such plates subjected to transient loadings.

A rather specialized category of laminate is a plate with unconstrained damping treatment, usually on one side of a substrate, so that the laminate has a total of two layers. The plate substrate is usually constructed of a homogeneous, isotropic material (although it could be of composite material), and the damping layer is usually a low-modulus, high-damping material such as an elastomer. Recent research into various kinds of damping treatments was surveyed by Nakra [106].

In a series of three papers, Ramachandra Reddy and his associates [107-109] analytically and experimentally investigated the response of plates with unconstrained damping layers to random acoustic excitation.

4.3 Sandwich Plates

A sandwich plate is one having a lightweight, flexible, relatively thick core (or several cores) with attached, stiff, relatively thin facings. Thus, it is customary to neglect transverse shear deformation in the facings and to include all of the transverse shear deformation in the core (or cores). The geometric configuration may be either symmetric or unsymmetric with respect to the midplane of the core.

Two papers were concerned with the effects of the in-plane inertia terms. Markus and Nanasi [110] made an extensive investigation of axisymmetric free vibration modes of clamped circular plates with isotropic facings and cylindrically orthotropic core. Grover and Kapur [111] analyzed the transient response of undamped, simply supported, rectangular plates subjected to a half-sine acceleration pulse. The materials were all isotropic; however, the facings were unsymmetric with respect to the mid-plane of the core. It was found that for relatively short loading durations, the effects of both rotatory and in-plane inertia actions were significant, while for relatively long shock durations, their effects were small.

Chonan [112] made a theoretical analysis of an infinitely long plate with thick facings and initial, in-plane, compressive stresses and subjected to a line load moving at constant speed. In another paper [113], the same author analyzed both axisymmetric and unsymmetric modes of circular annular plates with initial radial tension and elastically supported at both the inner and outer edges. Gupta and Jain [114] analyzed the free vibration of circular annular plates with linear radial thickness variation, using a cubic spline method.

Two papers were concerned with plate response to random acoustic loading. Ramachandra Rao and his associates [115] made a Galerkin-type analysis of clamped square panels with isotropic, elastic facings and isotropic, viscoelastic core. Reasonable agreement with results of experiments was obtained. Narayanan and Shanbhag analyzed the sound transmission and structural response of an ordinary isotropic sandwich panel [116] and one backed by an acoustic fluid cavity [117].

Ibrahim and his associates [118] made one of the few recent analyses of sandwich plates with facings of anisotropic material and arranged

unsymmetrically (incorrectly called unbalanced) with respect to the core middle plane. They considered simply supported rectangular plates with cross-ply and angle-ply facings.

In a series of two pioneering papers, Park and Bertoni [119,120] analyzed elastic wave propagation in a hexagonal-cell honeycomb core (not a complete sandwich). In the analysis they considered the discontinuous nature of the detailed honeycomb geometry. Thus, the waves considered were Bloch waves, which are analogous to plane waves in an elastic continuum. The purpose of their investigation was in support of the use of high-frequency waves for nondestructive evaluation of honeycomb cores and panels with honeycomb cores. However, in the light of the widespread use of hexagonal-cell cores in aircraft structural panels, their work should be of great interest to the structural dynamics community as well. In the first paper, their results showed that the honeycomb acts as an elastic continuum only at low frequencies. In the second paper they made a detailed study of the dispersive nature produced by the periodic nature of the structural configuration.

Torvik [121] recently reviewed the analysis and design of constrained-layer damping, i.e., a sandwich consisting of a flexible, high-damping core with thin, stiff, low-damping facings. For such structures with equally spaced discrete stiffeners, Slazak and Vaicaitis [122] developed an ingenious extension of the transfer-matrix method.

Recently, there has been considerable activity aimed toward the development of finite-element models of damped sandwich panels (or constrained-layer damping, if the reader prefers that terminology) as witnessed by [123-128].

5. NONLINEAR ANALYSES

Although nonlinear analyses obviously involve more computational effort than linear ones, the former are sometimes necessary, in these two instances in particular: (1) large-amplitude vibration (geometric nonlinearity) in which the slopes are sufficiently large to require the use of nonlinear strain-displacement relations, and (2) nonlinear constitutive equations.

5.1 Geometrically Nonlinear Thin Plates

Although the formulation of the equations governing the fundamental kinematic behavior of thin isotropic plates in the presence of geometric nonlinearity is generally attributed to von Kármán in 1910 [129], it was not brought into the formulation of laminated composites until the work of Whitney and Leissa in 1969 [130].

The most comprehensive work on the geometrically nonlinear analysis of both the static and dynamic behavior of thin plates is Chia's recent book [131]. Particularly pertinent here are Chapter 1 on nonlinear theory of laminated plates and Chapters 6 and 8 on postbuckling behavior and nonlinear flexural vibration of anisotropic plates and of unsymmetrically laminated anisotropic plates.

Table 1 is a summary of recent research papers [132-142] on geometrically nonlinear vibration of thin anisotropic and/or laminated plates. It should be mentioned that in [138], a specific investigation was made to illustrate the errors that can result from the use of Berger's approximation [143].

Table 1. Analysis of Large-Amplitude Vibration Thin Anisotropic and/or Laminated Plates

Investigator	Ref.	Planform Geometry	Matl.* Class	Type of Problem	Type of Solution
Alwar & Reddy, Nath & Alwar	132, 133	Circular	C0 with damping	Transient loading	Chebyshev polynomials in space; Houbolt numerical integration in time
Kanaka Raju & Venkateswara Rao	134	Circular (radially tapered thickness)	C0	Free vib.	Finite-element method
Venkateswara Rao & Kanaka Raju	135	Circular	C0	Axisymmetric free vib.	Finite-element method
Kunukkasseril & Venkatesan	136	Circular	Laminated isotropic	Axisymmetric free vib.	Galerkin
Chia & Sathyamoorthy	137	Parallelogram	R0	Free vib.	Galerkin
Prathap & Varalan	138	Parallelogram	R0	Free vib.	Galerkin
Sathyamoorthy	139	Elliptic	R0	Free vib.	Galerkin (combined deflection and stress-function formulation)
Sathyamoorthy & Chia	140	Elliptic	R0	Free vib.	Explicit polynomials (three displacement formulation)
Mei & Wentz	141	Rectangular	Laminated anisotropic w/damping	Random acoustic loading	Galerkin
Niyogi & Meyers	142	Rectangular	R0	Free vib.	Perturbation

*The symbols C0 and R0 denote cylindrically orthotropic and rectangularly orthotropic, respectively.

5.2 Geometrically Nonlinear Plates with Thickness Shear Flexibility

Recently, there has been considerable analytical research on plates which are geometrically nonlinear and yet have thickness shear deformation and rotatory inertia. This research, however, has emanated from only five research teams. The most extensive work was by Sathyamoorthy, either alone [144-149] or co-authored with Chia [150-153]. In these investigations of a variety of planform geometries, the same general methodology was used throughout: Galerkin solution in conjunction with Runge-Kutta numerical integration. No laminated plates were considered, although both specially orthotropic and generally orthotropic ones were; see Table 2 for details. It is interesting to note that in [146-148], Sathyamoorthy investigated the use of the Berger hypothesis [143] and found that it did not result in more than about 5% error for the cases investigated. Wang and Wang [154] used the Galerkin method and the method of multiple scales.

The most extensive finite-element work was by Reddy and his co-workers [155-160]. They investigated a variety of material classes and both free vibration and transient loading, as can be seen in Table 3. Also see [161-162].

5.3 Plates with Nonlinear Material Behavior

Due to the mathematical complexities involved, there have been very few analyses of composite panels with nonlinear material behavior. Katsaitis [163] considered the sinusoidally forced vibration of a clamped, rectangular, thin plate of specially orthotropic material having nonlinear damping. Ni [164] used a quadrilateral finite-element approach based on the finite-difference energy method to analyze thin panels constructed of materials with a cubic nonlinearity in in-plane shear deformation. Geometric nonlinearity was also included.

Table 2. Galerkin-Type Analyses of Large-Amplitude Vibration of Thick Anisotropic Plates

Investigator	Ref.	Planform Geometry	Matl. Class*	Flexural Boundary Conditions
Sathyamoorthy	144	Rectangular	R0	Simply supported
Sathyamoorthy	145-146	Parallelogram	R0	Clamped
Sathyamoorthy	147	Circular	R0	Clamped
Sathyamoorthy	148-149	Elliptic	R0	Clamped
Sathyamoorthy & Chia	150	Circular	R0	Clamped
Sathyamoorthy & Chia	151-152	Parallelogram	G0	Various
Sathyamoorthy & Chia	153	Rectangular	G0	Various
Wang & Wang	154	Rectangular	R0	Clamped & simply supported

*The symbols G0 and R0 denote generally orthotropic and rectangularly orthotropic, respectively.

Table 3. Finite-Element Analyses of Large-Amplitude Vibration of Thick Anisotropic and/or Laminated Plates

Investigator	Ref.	Planform Geometry	Matl. Class*	Type of Problem
Reddy, Huang, & Singh	155	Circular	C0	Free vib. (axisymmetric modes)
Reddy & Huang	156	Circular annular	C0	Free vib.
Reddy & Chao	157	Rectangular	R0	Free vib.
Reddy & Chao	158	Rectangular	LA	Free vib.
Reddy	159	Rectangular w/rectangular cutout	LA	Free vib.
Reddy	160	Rectangular	LA	Transient
Kanaka Raju et al.	161	Rectangular	R0	Free vib.
Mota Soares et al.	162	Rectangular	R0	Transient

*The symbols C0, R0, and LA denote cylindrically orthotropic, rectangularly orthotropic, and laminated anisotropic, respectively.

In a series of two reports [165-166], Zak and Pillasch used a quadrilateral finite element to model the transient behavior of laminated plates with geometric nonlinearity and orthotropic elastic-viscoplastic material behavior. The numerical time integration was accomplished by a finite-difference technique.

6. TRENDS AND SUGGESTIONS FOR FUTURE RESEARCH

These trends are notable in the research reviewed here:

- Increased emphasis on experimental research
- Considerable increase in the number of analyses including thickness shear flexibility
- Continued expansion of the use of the finite-element method, especially in the analysis of nonlinear problems

The author believes that the following aspects need to be investigated more fully in the future:

- Analyses of plates under transient loading
- Expanded attention to more realistic material models, including nonlinear stress-strain relations and material damping [167]
- Analyses of geometrically nonlinear panel flutter
- Interaction between vibration loading and material flaws, including fatigue crack propagation
- Study of the effects of laminate residual stresses, due to thermal-expansion mismatch, on vibration response of laminated plates
- Increased attention to material and laminate optimization, including hybrid composites

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Free and forced vibration, panel flutter, and impact are considered.

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